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Gracefulness of Joining Complete Bipartite Graphs Having A Common Vertex Set

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Abstract: In this paper we have shown that the join of two or more complete bipartite graphs having one vertex set in common is graceful. The graph obtained by joining two or more $\overline{K}_m$ graphs with the path $P_n$is also proved to be graceful.

Keywords: Graceful labeling, Complete bipartite graph.

1. INTRODUCTION

J.A. Gallian[1] has given a dynamic survey of graph labeling. M. Maheo[2] has shown that the book B_{2n} has an α-labeling for all $n \geq 1$. J.A. Gallian and D.S. Jungreis[3] have discussed the labeling of stacked books. In this present paper we have fixed our focus on the join of the complete bipartite graphs $K_{m,n}$ (m, fixed) for any n and proved the resultant graph to be graceful. We have also discussed the join of two or more $\overline{K}_m$ graphs with a path $P_n$and proved it to be graceful.

Definition 1.1 A graph $G = (V(G), E(G))$ with p vertices and q edges is said to admit *graceful labeling* if$f : V(G) \rightarrow \{0,1,2, \dots, q\}$ such that distinct vertices receive distinct numbers and$\{|f(u) - f(v)|/uv \in E(G)\} = \{1,2,3, \dots, q\}$.

2. MAIN RESULTS

Theorem 2.1 The join of complete bipartite graphs having one vertex set in common is graceful.

Proof: Consider two or more complete bipartite graphs say $K_{m,n}$ where m is same for all complete bipartite graphs whereas n can take different values. All the complete bipartite graphs under consideration are joined such that the complete bipartite graphs have one vertex set in common. Let the resultant graph have p vertices and q edges. The vertices in the common vertex set are denoted by $v_0, v_1, v_2, \dots, v_k$ where $k = m - 1$. The other vertices are denoted by $u_1, u_2, u_3, \dots, u_l$ where $l = p - m$. This larger graph is labeled gracefully by defining a function f as follows: $f: V(G) \rightarrow \{0,1,2, \dots, q\}$ such that

$$f(v_i) = i, \qquad for\ 0 \leq i \leq k$$

$$f(u_i) = i(k+1), for\ 1 \leq i \leq l$$

By using this definition of f the resultant graph can be labeled gracefully.

Illustration 2.2 Consider the complete bipartite graphs $K_{4,7}, K_{4,5}, K_{4,5}, K_{4,3}, K_{4,6}$and join them together with the common vertex set. This graph is gracefully labeled by using Theorem-1 and is shown in Figure-1.

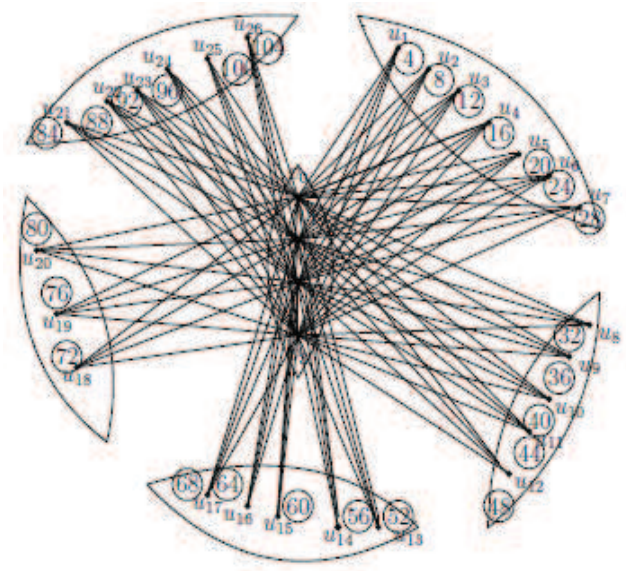

Figure1. Join of $K_{4,7}$, $K_{4,5}$, $K_{4,5}$, $K_{4,3}$ and $K_{4,6}$

The vertices v_0, v_1, v_2 and v_3 are labeled as $0, 1, 2$ and 3. The edges incident with v_0 are labeled as $4, 8, 12, 16, 20, 24, 28, 32,$
$36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80,$
$84, 88, 92, 96, 100$ and 104; The edges incident with v_1 are labeled as $3, 7, 11, 15,$
$19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59,$
$63, 67, 71, 75, 79, 83, 87, 91, 95, 99$ and 103; The edges incident with v_2 are labeled as $2, 6, 10, 14, 18, 22, 26, 30, 34,$
$38, 42, 46, 50, 54, 58, 62, 66, 70, 74, 78,$

$82, 86, 90, 94, 98$ and 102; The edges incident with v_3 are labeled as $1, 5, 9, 13,$
$17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57,$
$61, 65, 69, 73, 77, 81, 85, 89, 93, 97$ and 101.

Theorem 2.3The graph obtained by joining the graphs $\overline{K}_m$(complementary of K_m) for different values of m with the path P_n is graceful.

Proof:Consider a path P_n and two or more graphs $\overline{K}_m$ for different m. The larger graph is obtained by joining all the vertices of $\overline{K}_m$ to each and every vertex of the path. The resultant graph consists of p vertices and q edges. We denote the vertices of the path graph as $v_0, v_1, v_2, \dots, v_k$ where $k = n - 1$. The vertices of all $\overline{K}_m$ are denoted as $u_1, u_2, u_3, \dots, u_l$ where $l = p - n$. This resultant graph can be labeled gracefully by defining a function f as follows: $f: V(G) \to \{0,1,2, \dots, q\}$ such that

$f(v_0) = 0$

for $1 \leq i \leq k$,

$$f(v_i) = \begin{cases} \dfrac{i}{2}, & for\ even\ i \\ (n-1) - \left(\dfrac{i-1}{2}\right), & for\ odd\ i \end{cases}$$

$f(u_1) = 2n - 1$

for $2 \leq i \leq l$,

$f(u_i) = f(u_{i-1}) + n$

By using the definition of f, we label the graph gracefully.

Illustration 2.4 Consider the graph obtained by joining the path P_4 with $\overline{K}_3$, $\overline{K}_5$and$\overline{K}_4$. We label this graph gracefully by using Theorem-2 and is shown in Figure-2.

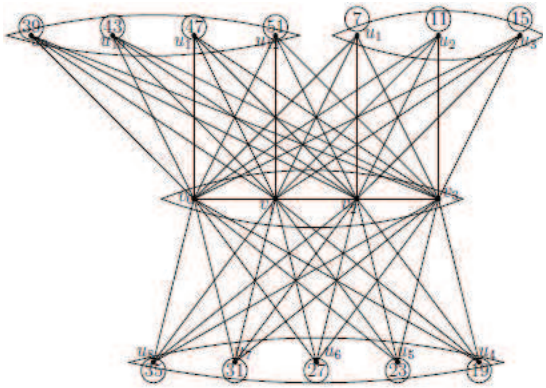

Figure 2 Join of P_3 with $\overline{K}_3, \overline{K}_5$ and $\overline{K}_4$

The vertices v_0, v_1, v_2 and v_3 are labeled as $0, 3, 1$ and 2. The edges v_0v_1, v_1v_2 and v_2v_3 are labeled as $3, 2$ and 1. The edges incident with v_0 are labeled as $7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47$ and 51; the edges incident with v_1 are labeled as $4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44$ and48; the edges incident with v_2 are labeled as $6, 10, 14, 18, 22, 26, 30, 34, 38, 42, 46$ and 50 and the edges incident with v_3 are labeled as $5, 9, 13, 17, 21, 25, 29, 33, 37,$ $41, 45$ and 49.

3. CONCLUSION:THE JOIN OF TWO OR MORE

Complete bipartite graphs at their common vertex set is graceful.We have proved that the graph obtained by joining two or more $\overline{K}_m$ graphs with path P_n is also labeled gracefully.

4. REFERENCES

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